Inferentialist Resource Semantics

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Mathematical Foundations of Programming Semantics
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Outline

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What is a ‘system’?

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Example

There are many including, for example, hospitals, universities, computers, communication networks (e.g., the internet), and more.

Example: Vending Machine

- *locations*: customer, vending machine

Figure: Reykjavík Univsersity

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Example: Vending Machine

- *locations*: customer, vending machine
- *resources*: money (i.e., kr in Iceland), chocolate bars
- *processes* (@C): 200kr is consumed, 1 chocolate bar is produced.

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- we intend to express all kinds of processes relevant to the domain
- we require accounting for counting, composition, comparison, sharing, and separation of resources

Current Resource Interpretations

Example (Resource Interpretations)

- number-of-uses interpretation of linear logic — proof-theoretic

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Hence, we need a unified approach!

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We desire a judgment

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Modelling II

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It should be interpreted as follows:

> *If policy Γ were to be executed with contextual resource $S(\cdot)$ based on the model $\mathscr{B}$, then the result state would satisfy φ.*

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To this end, we use **proof-theoretic semantics**.

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Proof-theoretic Semantics

A mathematical formulation of inferentialism based on modern formal notions of proof — e.g., natural deduction and sequent calculi.

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Soundness & Completenes

The word 'proof' here refers to a *pre-logical* notion of proof. Thus, the relationship between semantics and provability remains the same as it has always been: soundness and completeness are desirable features of formal systems.

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While B-eS sounds like Kripke semantics, it is **not**. Its relationship is an open problem. Let's talk later!

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- Validity in $\mathscr{B}$ is defined in the standard way
- Clauses include the following:

$$\begin{array}{lll r}
\Vdash_{\mathscr{B}} p & \text{iff} & \vdash_{\mathscr{B}} p & (\text{Atom}) \\
\Vdash_{\mathscr{B}} \varphi \wedge \psi & \text{iff} & \Vdash_{\mathscr{B}} \varphi \text{ and } \Vdash_{\mathscr{B}} \psi & (\wedge) \\
\Vdash_{\mathscr{B}} \varphi \to \psi & \text{iff} & \varphi \Vdash_{\mathscr{B}} \psi & (\to) \\
\Gamma \Vdash_{\mathscr{B}} \varphi & \text{iff} & \forall \mathscr{C} \supseteq \mathscr{B} (\Vdash_{\mathscr{C}} \Gamma \implies \Vdash_{\mathscr{C}} \varphi) & (\text{Inf})
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- $p \vdash_{\mathscr{B}} p$ should hold, but
- $p, q \vdash_{\mathscr{B}} p$ should not.

This is a straightforward modification! We elide the details to progress with the modelling — see the paper.

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Here are some clauses:

$$\begin{array}{rll}
\Vdash^{S}_{\mathscr{B}} \varphi \otimes \psi & \text{iff} \quad \forall \mathscr{C} \supseteq \mathscr{B} \, \forall T \, \forall p \, (\varphi \fatsemi \psi \Vdash^{T}_{\mathscr{B}} p \implies \Vdash^{S \fatsemi T}_{\mathscr{B}} p) & (\otimes) \\
\Vdash^{S}_{\mathscr{B}} \varphi \multimap \psi & \text{iff} \quad \varphi \Vdash^{S}_{\mathscr{B}} \psi & (\multimap) \\
\Gamma \Vdash^{S}_{\mathscr{B}} \varphi & \text{iff} \quad \forall \mathscr{C} \supseteq \mathscr{B} \, \forall T \, (\Vdash^{T}_{\mathscr{C}} \Gamma \implies \Vdash^{S \fatsemi T}_{\mathscr{C}} \varphi) & (\mathrm{Inf}) \\
\Vdash^{S}_{\mathscr{B}} \Gamma_1 \fatsemi \Gamma_2 & \text{iff} \quad \exists T_1, T_2 (S = T_1 \fatsemi T_2, \Vdash^{T_1}_{\mathscr{B}} \Gamma_1 \text{ and } \Vdash^{T_2}_{\mathscr{B}} \Gamma_2) & (\fatsemi)
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This is all quite intuitive — e.g., $(\otimes)$ recalls $\otimes_E$,

$$
\dfrac{\varphi \otimes \psi \qquad \begin{matrix}[\varphi \mathbin{⨟} \psi] \\ p\end{matrix}}{p}\ \otimes_E
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$$\Vdash^{S}_{\mathscr{B}} \varphi \mathrel{-\!\!*} \psi \quad \text{iff} \quad \varphi \Vdash^{S,(\cdot)}_{\mathscr{B}} \psi \qquad (\mathrel{-\!\!*})$$

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In general, for the base-extension semantics for some logic — e.g., IPL, ILL, BI:

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This admits the kind of resource semantics we desire. Recall:

- φ is an assertion describing (a possible state of) the system

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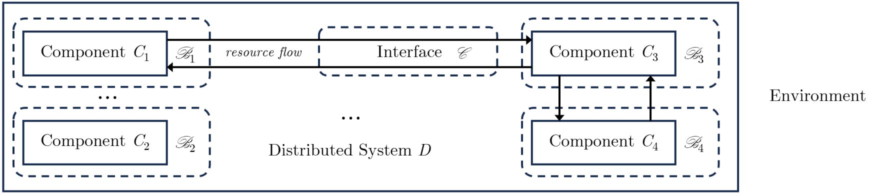

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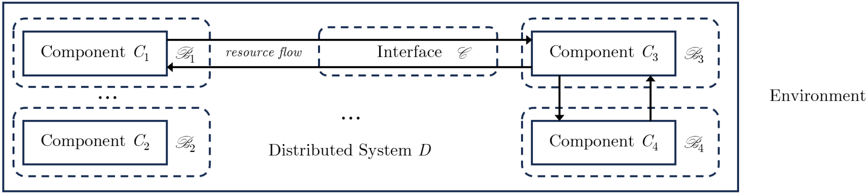

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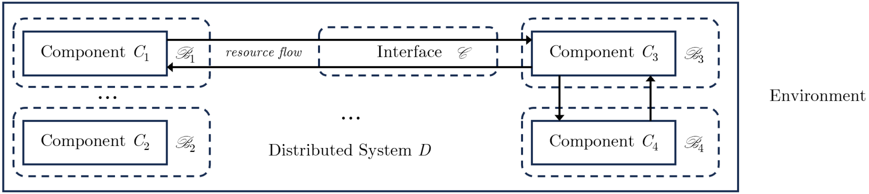

- describe each component C_i by a formula φ_i — this is its *policy*
- its model is given by a base $\mathscr{B}_i$ and resources S such that $\Vdash^{S}_{\mathscr{B}_i} \varphi_i$

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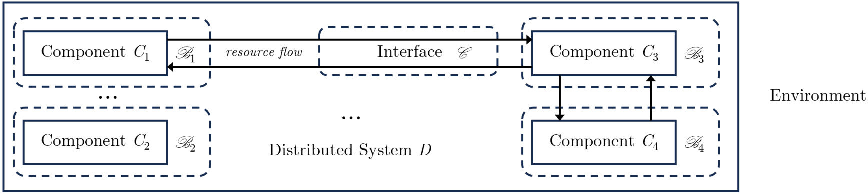

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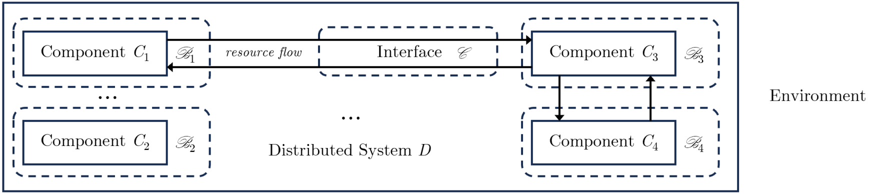

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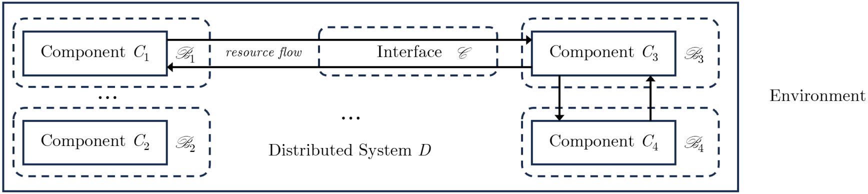

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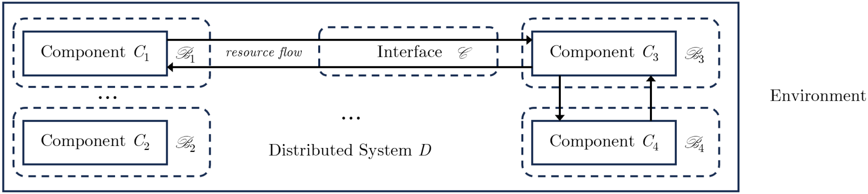

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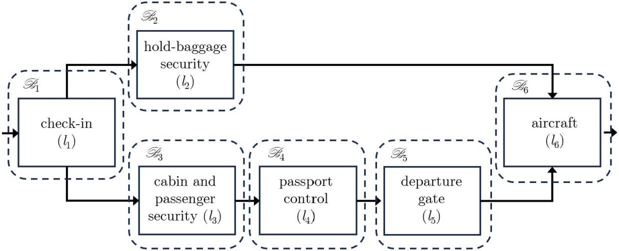
Remark. This approach to modelling is both *compositional* and *substitutional*.

Example: Airport Security modelled in BI



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Arriving with a valid ticket t and passport p is modelled by $\mathscr{B}$ such that $\Vdash_{\mathscr{B}}^{p \fatsemi t} \varphi$ — for more details, see our paper.

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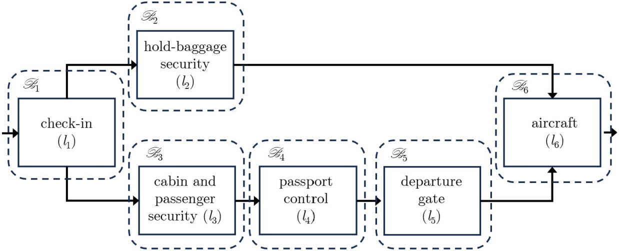

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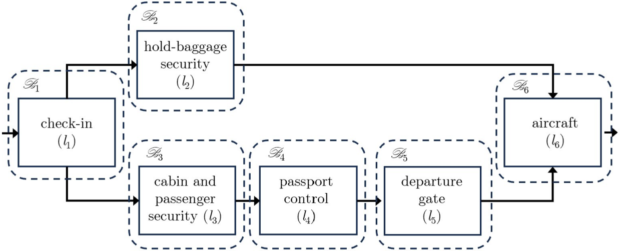

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Thesis

The paradigm of 'proof-theoretic semantics' provides an account of resource semantics that uniformly encompasses both the number-of-uses and sharing/separation interpretations of logics.

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- Future work: formalize the method and construct some useful models!